Noise Analysis of Non-Autonomous Radio Frequency Circuits

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Abstract

In this paper we consider the important problem of noise analysis of non-autonomous nonlinear RF circuits in presence of input signal phase noise. We formulate this problem as a stochastic differential equation and solve it in the presence of circuit white noise sources. We show that the output noise of a nonlinear non-autonomous circuit, driven by a periodic input signal with phase noise, is stationary and *not* cyclostationary (as would be predicted by traditional analyses). We also show that effect of input signal phase noise is to act as additional *white* noise source. This result is derived using a full nonlinear analysis of the problem and cannot be predicted by traditional linear analysis based techniques. Input signal phase noise can be an important portion of the overall output noise of the non-autonomous circuit. In our opinion, existing analyses have not considered this effect in a rigorous manner. We also relate this solution to results of the existing nonlinear time domain and frequency domain methods of noise analysis and point out the modifications required for the present techniques. We illustrate our technique using an example.

1 Introduction

In high speed communication, instrumentation and signal processing applications, random electrical noise that emanates from devices has a direct impact on critical high level performance metrics, e.g., bit error rate or signal to noise ratio, blocking performance, spectral leakage. Hence predicting noise in such systems at the design stage is extremely important. RF circuits are usually analyzed for their steady state behaviour under one or more periodic excitations. In a typical RF system, some amplifiers and other analog circuits such as mixers, filters and oscillators do not operate in small signal condition. These circuits usually have one or more large signal time varying inputs which cause the statistics of the circuit noise sources to be time varying. Hence stationary noise analysis techniques are inadequate for analyzing the noise behaviour of such circuits since they do not capture important nonlinear aspects such as frequency translation of noise spectra.

The problem of predicting noise performance of nonlinear RF circuits, being an important one, has attracted the attention of several researchers. There are several techniques to model the time varying nature of noise and the response of the circuit to such noise (see Section 2). However, these techniques make assumptions about the nature of input signals, specifically their noise characteristics which are not rigorous. These techniques conclude that, in the presence of periodically varying inputs, the circuit noise statistics are also periodically time varying.

This paper addresses the problem of formulating and solving non-autonomous circuit equations in presence of a noisy input signal (which is derived from a *real* oscillator). It has been shown [1] that it is not mathematically rigorous to view the noisy oscillator output as a deterministic signal with additive phase and amplitude noise since linear perturbation analysis is not valid for oscillators. A mathematically consistent representation of the oscillator output is a sum two wide sense stationary stochastic processes: a large signal output process with phase deviation which has the statistics of a Wiener process (Brownian motion) and a "small" amplitude noise process. The resulting oscillator output has a Lorentzian spectrum. We use this correct representation of oscillator output noise in our analysis. We show that:

- The output of nonlinear non-autonomous systems in the presence of period input with Brownian motion phase deviation, is asymptotically wide sense stationary.
- The Lorentzian spectrum of the input signal and the characteristics of the Brownian motion input phase deviation process are preserved at the output.
- Noisy input is shown to contribute a wide-band amplitude noise term at the output of the nonlinear circuit. This appears as a white noise source modulated by the time derivative of the steady state response of the system.
- This result is generalized to case of multi-tone inputs. For case of k large periodic inputs, k additional white noise sources (suitably modulated) need to be considered.

The intuition behind these results that the non-autonomous system in conjunction with the driving oscillator can be viewed as a composite large oscillator. Hence the observations made in [1] about the output of the noisy oscillator carry over to this composite system which is also autonomous. However, for the non-autonomous circuits (such as mixers) the output signal frequency typically is far away from the input signal frequency (or any harmonics of that). Hence noise due to the phase deviation process in the frequency range of interest is small, compared to the wide band amplitude noise process. Hence, we need to compute the amplitude noise process in this case.

The rest of the paper is organized as follows: We begin by reviewing some of the existing techniques for noise analysis of non-autonomous circuits (Section 2). In Section 3 we introduce some basic mathematical notation about non-autonomous system of equations. We begin our analysis (Section 4) by briefly reviewing circuit noise equations in presence of a deterministic large periodic input signal. We then analyze the noiseless circuit with input signal phase noise only and show that the general noise analysis case is an extension of this. Finally (Section 5) we demonstrate our technique with an example.

2 Previous Works

Several techniques have been proposed, both in the time domain [2, 3, 4] and in the frequency domain [5, 6], for predicting noise performance for nonlinear circuits. Both these classes of techniques take advantage of the fact that for most RF applications, the circuit is driven by periodic (or *quasi*-periodic) signals. Hence only the steady state performance of the circuit over a small time interval, usually over one period of the input signal is sufficient to describe its behaviour. Noise analysis is performed by linearizing the circuit around its time varying response to the large signal. The underlying assumption is that small perturbations, deterministic or stochastic, result in small deviations in the response of the circuit, leading to additive noise in the case of stochastic perturbations. This assumption is rigorously justified for stable non-autonomous systems in the presence of large deterministic signals. If the noisy input signal can be represented as an additive noise over and above the deterministic periodic signal, the small deviation assumption is again justified. The input signal noise can be assumed to be a circuit noise source with equivalent statistics, at the input node. These techniques conclude that the circuit noise statistics are periodically time varying in presence of periodic inputs. However, [7] conclude that only the stationary component of this noise is important without giving a mathematical justification. The popular use of noise figure of the individual blocks in a receiver path to compute its overall noise performance also assumes that the output noise is stationary. On the other hand, [5] conclude that this notion of noise figure is not sufficient to characterize the output noise and resort to computing the full cyclostationary statistics at the output of a block.

However all these approaches do not explicitly consider the effect of phase noise in input signal. Also, it is not mathematically rigorous to view the oscillator output as a deterministic signal with additive phase and amplitude noise [1]. Hence approaches that perform stationary/cyclostationary noise analysis of nonlinear RF driven systems (e.g. mixers etc.) in presence of *deterministic* periodic signals need to be re-examined. This paper addresses precisely this problem.

3 Mathematical Preliminaries

The dynamics of a unperturbed non-autonomous system can be described by the following system of differential equations

$$\dot{x} = f(x) + b_0(t) \qquad (3.1)$$

where $x \in \mathbb{R}^n$ is a vector of state variables, $f(x) : \mathbb{R}^n \to \mathbb{R}^n$ and $b_0(t) : \mathbb{R} \to \mathbb{R}^n$ is deterministic T-periodic input. We assume that this equation satisfies the Cauchy-Peano existence and uniqueness theorem for the initial value problem [8]. We further assume that the system is stable and non-autonomous in the sense that in the absence of $b_0(t)$, the steady state solution of this equation is 0. We assume that the steady state solution of this system (in presence of $b_0(t)$) is given by $x_s(t)$, which is also periodic with period T. This assumption is justified for almost all non-autonomous RF components except frequency dividers where the output is periodic with a larger period T'. The analysis we present here is therefore not valid for frequency dividers.

We are interested in the response of this system in the presence of noise, both in the form of circuit intrinsic noise $D(x)\xi(t)$ and phase noise in the input signal of the form $b_0(t+\alpha(t))$ where $D(\cdot) : \mathbb{R}^n \to \mathbb{R}^{n\times p}$ describes the connectivity and modulation of the noise sources, $\xi(\cdot) : \mathbb{R} \to \mathbb{R}^p$ are white noise sources and $\alpha(\cdot) : \mathbb{R} \to \mathbb{R}$ is the phase deviation process of the input signal which is a scaled Brownian motion process, i.e., of the form $\sqrt{c}B(t)$ [1] where c is the rate of increase of the variance. Hence the modified system is governed by the following differential equation.

$$\dot{x} = f(x) + b_0(t+\alpha(t)) + D(x)\xi(t)$$

or equivalently, in stochastic differential equation form as

$$\mathrm{d}x = f(x)\mathrm{d}t + b_0(t+\alpha(t))\mathrm{d}t + D(x)\mathrm{d}B_p(t) \qquad (3.2)$$

where $B_p(t)$ is a p-dimensional Brownian motion[1].

4 Noise Analysis of Non-Autonomous Systems

We begin with a brief review of cyclostationary noise analysis.

4.1 Cyclostationary Approach

Consider the above system of equations (3.2) but with ideal input source signal $b_0(t)$, i.e.,

$$\mathrm{d}x_s = f(x)\mathrm{d}t + b_0(t)\mathrm{d}t + D(x)\mathrm{d}B_p(t) \qquad (4.1)$$

Assume that the perturbed response of this system is $x_s(t)+y(t)$ where $y(t)$ is the small stochastic deviation of the response of the system. Substituting this in (4.1) we have

$$\begin{aligned}\mathrm{d}x_s(t) + \mathrm{d}y(t) = {} & f\big(x_s(t)+y(t)\big)\mathrm{d}t + b_0(t)\mathrm{d}t \\ & + D\big(x_s(t)+y(t)\big)\mathrm{d}B_p(t)\end{aligned}$$

Linearizing $f(x_s(t)+y(t))$ around $x_s(t)$, ignoring $y(t)$ in the argument of $D(\cdot)$ and using that fact that $x_s(t)$ satisfies (3.1), the above equation reduces to

$$\mathrm{d}y(t) \approx \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x_s(t)} y(t)\mathrm{d}t + D(x_s(t))\mathrm{d}B_p(t) \qquad (4.2)$$

where $\left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x_s(t)} = J(t)$ is the Jacobian of $f(x)$ evaluated at $x_s(t)$. Since $x_s(t)$ is T-periodic, it follows that $J(t)$ is also T-periodic. Since $D(x_s(t))$ is also T-periodic, and the system of equations is

[1]For sake of simplicity, we use the state equation formulation to describe the system. These results and techniques can be extended to the mixed differential-algebraic equation formulation (for instance, as in modified nodal analysis (MNA)) of the form $\frac{\mathrm{d}q(x)}{\mathrm{d}t} + f(x) = 0$ in a straightforward manner.

linear in $y(t)$, the above system of equations describes a linear periodic time-varying system of equations governing the deviation of the circuit response and $y(t)$ is also cyclostationary. The time-varying statistics of $y(t)$ are usually computed by considering the periodic time-varying noise as an input to a linear periodically time-varying system corresponding to (4.2) which is computed directly from the steady state response of the circuit [5].

4.2 Response to Input Signal Phase Noise

We now introduce our approach to solving (3.2). To illustrate the basic principles we will assume that the nonlinear circuit itself is noiseless, i.e., $D(x) = 0$. We will relax this assumption later. As indicated earlier, the additive amplitude noise component of the input signal can also be absorbed in the circuit equations so we will only consider an input signal which has phase deviation but no amplitude noise, i.e., of the form $b_0(t+\alpha(t))$. Hence (3.2) reduces to $\dot{x} = f(x) + b_0(t+\alpha(t))$. Equivalently

$$\mathrm{d}x = f(x)\mathrm{d}t + b_0(t+\alpha(t))\mathrm{d}t \tag{4.3}$$

where as before $\alpha(t) = \sqrt{c}B(t)$. Assuming that c is small, i.e., the input signal phase noise is small and the system is stable, the response of the system is of the form

$$x_s(t+\alpha(t)) + y_1(t) \tag{4.4}$$

where $y_1(t)$ is assumed to be small. By choosing the response of this form, we are assuming that the circuit is able to follow any variations in instantaneous input frequency. This is a valid assumption if input signal phase noise (i.e., c) is assumed to be small and the nonlinear circuit is stable (non-oscillatory). We first make the following useful observations:

Definition 1 *Define* $s(t) = t + \alpha(t)$.

Lemma 4.1 $s(t)$ *as defined in Definition 1 is an* Itô process *[9].*

The advantage of proving $s(t)$ to be an Itô process is that we can use the following result from [9] to evaluate $\mathrm{d}x_s(t+\alpha(t))$.

Lemma 4.2 (The Itô Formula [9]) *Let* $X(t)$ *be an Itô process given by* $\mathrm{d}X(t) = u\mathrm{d}t + v\mathrm{d}B(t)$ *where* $B(t)$ *is Brownian motion. Let* $g(t,x)$ *be twice continuously differentiable on* $\mathbb{R}_+ \times \mathbb{R}$. *Then* $Y(t) = g(t, X(t))$ *is also an Itô process and*

$$\begin{aligned}\mathrm{d}Y(t) = &\frac{\partial g}{\partial t}(t,X(t))\mathrm{d}t + \frac{\partial g}{\partial x}(t,X(t))\mathrm{d}X(t)\\ &+ \frac{1}{2}\frac{\partial^2 g}{\partial x^2}(t,X(t))(\mathrm{d}X(t))^2\end{aligned}$$

where $(\mathrm{d}X(t))^2 = \mathrm{d}X(t)\mathrm{d}X(t)$ *is computed according to the rules* $\mathrm{d}t\mathrm{d}t = \mathrm{d}t\mathrm{d}B(t) = 0$ *and* $\mathrm{d}B(t)\mathrm{d}B(t) = \mathrm{d}t$.

Let $\dot{x}_s(t) = \frac{\mathrm{d}x_s}{\mathrm{d}t}$ and $\ddot{x}_s(t) = \frac{\mathrm{d}^2x_s}{\mathrm{d}t^2}$. Then

$$\begin{aligned}\mathrm{d}x_s(s(t)) &= \dot{x}_s(s(t))\mathrm{d}s(t) + \frac{1}{2}\ddot{x}_s(s(t))[\mathrm{d}s(t)]^2\\ &= \dot{x}_s(s(t))(\mathrm{d}t + \sqrt{c}\mathrm{d}B(t)) + \frac{c}{2}\ddot{x}_s(s(t))\mathrm{d}t\end{aligned}$$

Substituting the form of the solution (4.4) and the above expansion in (4.3) and linearizing $f(x)$ around $x_s(s(t))$ we obtain

$$\begin{aligned}&\mathrm{d}y_1(t) + \dot{x}_s(s(t))(\mathrm{d}t + \sqrt{c}\mathrm{d}B(t)) + \frac{c}{2}\ddot{x}_s(s(t))\mathrm{d}t\\ &\approx f(x_s(s(t)))\mathrm{d}t + \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x_s(s(t))} y_1(t)\mathrm{d}t + b_0(s(t))\mathrm{d}t\end{aligned}$$

Since $x_s(t)$ is the steady state solution of (3.1),

$$\frac{\mathrm{d}x_s}{\mathrm{d}t}(s(t)) = f(x_s(s(t))) + b_0(s(t))$$

and hence

$$\mathrm{d}y_1(t) = J(s(t))y_1(t)\mathrm{d}t + M_1(s(t))\mathrm{d}B(t) + M_2(s(t))\mathrm{d}t \tag{4.5}$$

where $M_1(t) = -\sqrt{c}\dot{x}_s(t)$ and $M_2(t) = -0.5c\ddot{x}_s(t)$ are also periodic with period T.

Remark:

- The term $M_1(s(t))\mathrm{d}B(t)$ represents a white noise source modulated by the time derivative of the steady state response. This means that phase noise in the input signal results in a time-varying wide-band noise at the output of the nonlinear circuit.
- The periodic coefficients J, M_1 and M_2 are evaluated at $s(t) = t + \alpha(t)$ and *not* at t.
- (4.5) is a stochastic differential equation which is linear in $y_1(t)$ and the terms $M_1(s(t))\mathrm{d}B(t)$ and $M_2(s(t))\mathrm{d}t$ represent two inputs to this linear system. Hence $y_1(t)$ can be represented as $y_{11}(t) + y_{12}(t)$ where $y_{11}(t)$ satisfies

  $$\mathrm{d}y_{11}(t) = J(s(t))y_{11}(t)\mathrm{d}t + M_1(s(t))\mathrm{d}B(t) \tag{4.6}$$

  and $y_{12}(t)$ satisfies

  $$\mathrm{d}y_{12}(t) = J(s(t))y_{12}(t)\mathrm{d}t + M_2(s(t))\mathrm{d}t \tag{4.7}$$

To solve (4.6) we make the following useful observations:

Definition 2 *Define* $U(t) = \sqrt{c}B(t) \mod T$

Lemma 4.3 *The solution of* (4.6) *is the same as the solution of*

$$\mathrm{d}y_{11}(t) = J(t+U(t))y_{11}(t)\mathrm{d}t + M_1(t+U(t))\mathrm{d}B(t)$$

Proof: Follows from the fact that $J(t)$ and $M_1(t)$ are T-periodic. ■

Lemma 4.4 *Asymptotically* $U(t)$ *is a random process which is uniformly distributed between 0 and* T *for every* t.

Proof: Follows from the fact that the variance of Brownian motion grows unbounded with t. ■

Definition 3 *Define* $r = t + U(t)$ *and* $z_{11}(r) = y_{11}(t)$.

Then using the fact that c is small, it follows that (4.6) is equivalent to the following equation

$$\mathrm{d}z_{11}(r) = J(r)z_{11}(r)\mathrm{d}r + M_1(r)\mathrm{d}B(r)$$

Note that this equation is in the exact same form as (4.2). This means that $z_{11}(r)$ is a cyclostationary process. Moreover, since $J(\cdot)$ is the Jacobian of a stable system, if $M_1(r)$ is small, $z_{11}(r)$ is small for all r. Hence the above analysis is consistent.

Using the fact that $y_{11}(t) = z_{11}(r) = z_{11}(t+U(t))$ and $U(t)$ is uniformly distributed between 0 and T for all t, we conclude that

Theorem 4.5

- *$y_{11}(t)$ is stationary*
- *The autocorrelation $\mathbb{E}\left[y_{11}(t)y_{11}^T(t+\tau)\right]$, where $y_{11}(t)$ is the solution of*

$$\mathrm{d}y_{11}(t) = J(t+\alpha(t))y_{11}(t)\mathrm{d}t + M_1(t+\alpha(t))\mathrm{d}B(t)$$

is the stationary component of $\mathbb{E}\left[z_{11}(t)z_{11}^T(t+\tau)\right]$ where $z_{11}(t)$ is the solution of

$$\mathrm{d}z_{11} = J(t)z_{11}(t)\mathrm{d}t + M_1(t)\mathrm{d}B(t)$$

Proof: [10] ■

Now we consider (4.7). Defining $z_{12}(s) = y_{12}(t)$ as before we conclude that $z_{12}(s)$ satisfies the following differential equation

$$\mathrm{d}z_{12} = J(s)z_{12}(s)\mathrm{d}s + M_2(s)\mathrm{d}s$$

Using the same arguments we can conclude that the steady state solution $z_{12}(s)$ of the above equation remains small and bounded for small c's. $z_{12}(s)$ is a periodic determinstic signal (except that s has an additive Brownian motion term).

Therefore, the output consists of a sum of three term, $x_s(t+\alpha(t))$, $y_{12}(t) = z_{12}(t+\alpha(t))$ and $y_{11}(t)$. Using same arguments as in [1] we conclude that $x_s(t+\alpha(t))$ and $y_{12}(t)$ are wide-sense stationary stochastic processs with Lorentzian spectrum around the frequency of input signal $b_0(t)$. Hence as indicated in Section 1 this typically contributes to noise power outside the ouput frequency band of interest. $y_{11}(t)$ is a wide band noise term which contributes to noise in the frequency band of interest. Next we extend this analysis to include circuit noise sources and discuss numerical computation of statistics of $y_{11}(t)$.

4.3 General Noise Analysis

We now consider (3.2). We assume that the response of the circuit is of the form $x_s(t+\alpha(t)) + y_0(t)$ Proceeding exactly as in the previous subsection, we conclude that $y_0(t) = y_{01}(t) + y_{02}(t)$ where $y_{01}(t)$ satisfies

$$\begin{aligned}\mathrm{d}y_{01}(t) = J(s(t))y_{01}(t)\mathrm{d}t + M_1(s(t))\mathrm{d}B(t) \\ + M_0(s(t))\mathrm{d}B_p(t)\end{aligned} \tag{4.8}$$

where $B(t)$ and $B_p(t)$ are uncorrelated and $M_0(t) = B(x_s(t))$. $y_{02}(t)$ is still given by (4.7). (4.8) can be rewritten as

$$\mathrm{d}y_{01}(t) = J(s(t))y_{01}(t)\mathrm{d}t + M(s(t))\mathrm{d}B_{p+1}(t)$$

where $M(t) = [M_1(t) \quad M_0(t)]$ and $B_{p+1}(t) = \begin{bmatrix} B(t) \\ B_p(t)\end{bmatrix}$. It follows that

Corollary 4.6

- *$y_{01}(t)$ is stationary*
- *The autocorrelation $\mathbb{E}\left[y_{01}(t)y_{01}^T(t+\tau)\right]$, where $y_{01}(t)$ is the solution of*

$$\mathrm{d}y_{01}(t) = J(t+\alpha(t))y_{01}(t)\mathrm{d}t + M(t+\alpha(t))\mathrm{d}B_{p+1}(t)$$

is the stationary component of $\mathbb{E}\left[z_{01}(t)z_{01}^T(t+\tau)\right]$ where $z_{01}(t)$ is the solution of

$$\mathrm{d}z_{01} = J(t)z_{01}(t)\mathrm{d}t + M(t)\mathrm{d}B_{p+1}(t)$$

Hence we conclude that we can still use the existing nonlinear noise simulation algorithms (such as [5]) for predicting noise in the non-autonomous nonlinear systems with a couple of modifications.

- We need to add another noise source to the noise equations corresponding to the phase to wide-band amplitude noise conversion of the input signal phase noise by the nonlinear system. For this we first need to perform noise analysis of the oscillator(s) to determine the phase noise performance of the input signal.
- We only need to consider the stationary component of the cyclostationary noise statistics computed by the algorithm.

4.4 Extension to Mutli-Tone Inputs

The above analysis can also be extended to the case when the non-autonomous circuit is driven by two or more large periodic signals with incommensurable frequencies. For simplicity consider the two tone case. Let the input $b_0(t)$ in (3.1) be a sum of two large signals $b_1(t) + b_2(t)$ which are periodic with periods T_1 and T_2. Then the steady state response $x_s(t)$ is *quasi*-periodic and can be written as [11]

$$x_s(t) = \sum_{i=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} X(i,k)\exp(ji\omega_1 t)\exp(jk\omega_2 t)$$

The bi-variate form [11, 12, 13] of $x_s(t)$ is given by

$$\hat{x}_s(t_1,t_2) = \sum_{i=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} X(i,k)\exp(ji\omega_1 t_1)\exp(jk\omega_2 t_2)$$

where $x_s(t) = \hat{x}_s(t,t)$. Appealing to above the bi-variate form of $x_s(t)$, the solution of $\dot{x} = f(x) + b_1(t+\alpha_1(t)) + b_2(t+\alpha_2(t)) + D(x)\xi(t)$ circuit equations with noise, ($\alpha_{1,2}(t) = \sqrt{c_{1,2}}B_{1,2}(t)$) can be assumed to be of the form

$$\begin{aligned}x_{two\ tone} = \sum_{i=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} X(i,k)e^{ji\omega_1(t+\alpha_1(t))}e^{jk\omega_2(t+\alpha_2(t))} \\ + y_1(t)\end{aligned}$$

Since

$$\mathrm{d}s(t) = \begin{bmatrix}1\\1\end{bmatrix}\mathrm{d}t + \begin{bmatrix}\sqrt{c_1} & 0 \\ 0 & \sqrt{c_2}\end{bmatrix}\begin{bmatrix}\mathrm{d}B_1(t)\\ \mathrm{d}B_2(t)\end{bmatrix}$$

is a two-dimensional Itô process [9], we can use the multi-dimensional Itô formula [9] to evaluate $\mathrm{d}x_{two\ tone}$. The rest of the analysis proceeds in a similar manner. We show that [14] the wide-band output noise of a non-sutonomous circuit driven by two large tones is given by the stationary component of $z_{01}(t)$ where $z_{01}(t)$ is governed by the following stochastic differential equation

$$\begin{aligned}\mathrm{d}z_{01}(t) = J(x_s(t))z_{01}(t)\mathrm{d}t + D(x_s(t))\mathrm{d}B_p(t) \\ - \sqrt{c_1}\left.\frac{\partial\hat{x}_s(t_1,t_2)}{\partial t_1}\right|_{t_1=t_2=t}\mathrm{d}B_1(t) \\ - \sqrt{c_2}\left.\frac{\partial\hat{x}_s(t_1,t_2)}{\partial t_2}\right|_{t_1=t_2=t}\mathrm{d}B_2(t)\end{aligned}$$

i.e., we now need to add *two* more white noise sources, one each corresponding to the input signals. This result immediatly generalizes to the case when the circuit is driven by more than two large periodic signals.

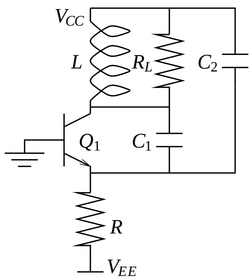

Figure 1: Colpitt's oscillator

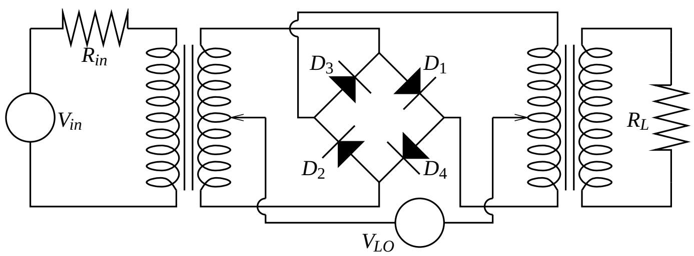

Figure 2: Four diode based mixer

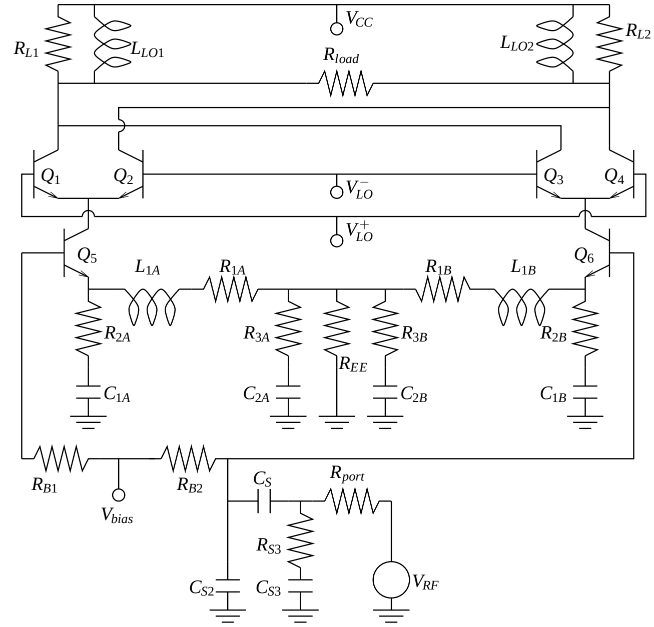

Figure 3: Gilbert cell based mixer

5 Experimental Results

The noise simulation algorithm is implemented in MATLAB. We use the time domain technique presented in [1] for performing noise simulation for oscillators and the harmonic balance based technique presented in [5] to perform the noise analysis of the non-autonomous portions of the circuit. The steady state response of the circuit and the Jacobians are computed by performing transient simulations in SPICE3 and later handed over to MATLAB.

We illustrate our technique using two examples. Consider the oscillator shown in Figure 1. The basic configuration is a Colpitts oscillator. This circuit has 11 state variables and 8 noise sources. c was computed to be 3.19×10^{-17} sec which corresponds to relative noise power of 78.1 dBc/Hz below the carrier at an offset frequency of 100 kHz. This oscillator is used to generate the 2.2 GHz LO which drives the passive and active mixers shown in Figure 2 and 3. The passive mixer has 21 state variables along with 10 noise source. The Gilbert cell based mixer has 53 state variables along with 46 noise sources (excluding the one added for the oscillator noise contribution). The RF signal is assumed to come from a 50 Ω port at 2.4 GHz. The noise figure of this mixer at the IF port at 200 MHz, without the contribution of the LO phase, noise was computed to be 13.3 dB and 5.5 dB for the passive and the active mixer repectively. Including the effect of LO phase noise, the noise figure increased to 9.15 dB for the active mixer while there was no increase in the passive mixer output noise.

Figure 4 shows the increase in noise figure for the active mixer (from the noiseless oscillator case) as a function of c for this circuit. This increase is negligible for $c < 1 \times 10^{-17}$ sec but as c increases beyond this value, the noise figure degrades rapidly. This cross-over point is the value of c where the input signal phase noise starts dominating over the circuit noise. This also suggests that for this particular mixer, it is an overkill for the LO to have phase noise performance better than 83 dBc/Hz at 100 kHz offset.

For the passive mixer, it is observed that as the input signal phase noise is increased, the output noise is not affected (noise at other

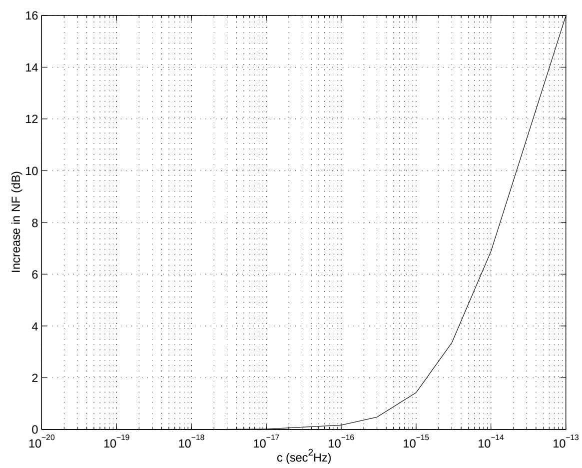

Figure 4: Increase of active mixer NF with input signal phase noise

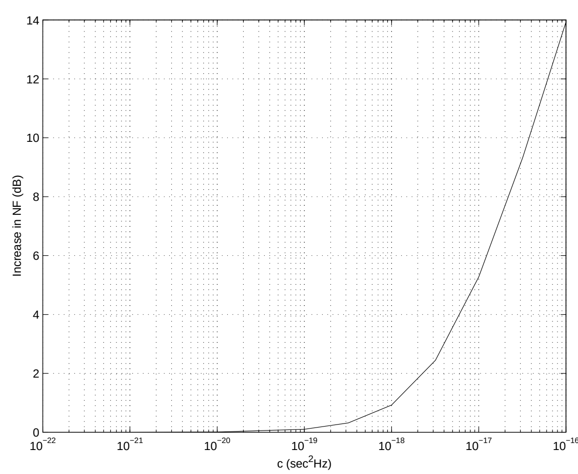

Figure 5: Increase of passive mixer (with 2% offset in the center tap of the output inductor) NF with input signal phase noise

circuit nodes *does* increase due to increasing input signal phase noise). This is explained by the perfect symmetry of the circuit. Since the circuit is symmetric, the white noise source due to input signal phase noise appears as a common mode signal to the output node of this circuit and hence does not affect the noise at the output node of the circuit. However, in a realistic scenario, perfect symmetry cannot be achieved. To mimic this, a 2% offset in the center tap of the output transformer was artificially introduced.

Figure 5 shows the increase in noise figure (from the noiseless oscillator case) as a function of c for the passive mixer with a 2% offset in the center tap of the output transformer. This increase is negligible for $c < 1 \times 10^{-19}$ sec but as c increases beyond this value, the noise figure degrades rapidly.

6 Conclusions

This paper addresses the problem of performing noise simulation for non-autonomous nonlinear circuits driven by large periodic signals which are themselves generated by oscillators and therefore have phase noise. We showed that noise at the output of these systems is stationary and that we can use a modified version of existing nonlinear noise simulation techniques to evaluate noise performance. We illustrated this technique with two examples.

From the experimental results, we conclude that input signal phase noise can be an important portion of the overall output noise of the non-autonomous circuit. In our opinion, existing analyses have not considered this effect rigorously. The effect of Brownian motion phase deviation in the input signal is to act as additional *white* noise source in the nonlinear non-autonomous circuit. This result is derived using a full nonlinear analysis of the problem and cannot be predicted by traditional linear analysis based techniques. This analysis determines precisely how much the output noise increases due to input signal phase noise. This is very important from a system design perspective, since performance constraints on one component (e.g., constraints on the phase noise performance of the LO signal) can be obtained from the description of the other (mixer noise performance). Since the output noise of a non-autonomous circuit can be fully characterized (including the effect of input signal phase noise), this model can used in a system level noise simulation methodology for complex RF systems. These models can also be used for behavioural level performance optimization and constraint generation for the RF components.

This technique, as presented here can only handle white noise sources. However for noise with long-term correlations, i.e., flicker noise, the steps outlines above are not rigorously justified. [3] used the modulated stationary noise model to analyze flicker noise. However, the asymptotic arguments in this formulation need to be carefully examined before these results can be carried over to the flicker noise case as well.

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